Scalar σ meson effects in radiative ρ^0-meson decays

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We study the radiative $\rho^0 \to \pi^+\pi^-\gamma$ and $\rho^0 \to \pi^0\pi^0\gamma$ decays and we calculate their branching ratios using a phenomenological approach by adding to the amplitude calculated within the framework of chiral perturbation theory and vector meson dominance the amplitude of σ-meson intermediate state. Our results for the branching ratios are in good agreement with the experimental values.

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The radiative decays of neutral vector mesons into a single photon and a pair of neutral pseudoscalar mesons have been a subject of continuous interest. The studies of such decays may serve as tests for the theoretical ideas about the nature of the intermediate states and the interesting mechanisms of these decays, and they may thus provide information about the complicated dynamics of meson physics in the low energy region.

The very recent measurement of the branching ratio for the decay $\rho^0 \to \pi^0\pi^0\gamma$ by the SND Collaboration obtained the value $\mathrm{BR}(\rho^0 \to \pi^0\pi^0\gamma) = (4.1^{+1.0}_{-0.9} \pm 0.3)\times 10^{-5}$ [1], thus improving their previous preliminary report of $\mathrm{BR}(\rho^0 \to \pi^0\pi^0\gamma) = (4.8^{+3.4}_{-1.8} \pm 0.3)\times 10^{-5}$ [2]. On the other hand, the branching ratio for the decay $\rho^0 \to \pi^+\pi^-\gamma$ was reported earlier by the Novosibirsk group as $\mathrm{BR}(\rho^0 \to \pi^+\pi^-\gamma) = (9.9 \pm 1.6)\times 10^{-3}$ [3,4], and it was observed that the pion bremsstrahlung is the main mechanism for this decay with the structural radiation proceeding through the intermediate scalar resonance making less than one-order of magnitude smaller contribution to the branching ratio [3].

The theoretical studies of radiative ρ-meson decays was initiated by Singer [5] who calculated the amplitude for the decay $\rho^0 \to \pi^+\pi^-\gamma$ by considering only the bremsstrahlung mechanism, and he assumed that the decay $\rho^0 \to \pi^0\pi^0\gamma$ proceeds through an $(\omega\pi)$ intermediate state as $\rho^0 \to (\omega)\pi^0 \to (\pi^0\gamma)\pi^0$. The vector meson dominance (VMD) calculation of Bramon *et al.* [6] with this intermediate state using standard Lagrangians obeying SU(3) symmetry gave the value $\mathrm{BR}(\rho^0 \to \pi^0\pi^0\gamma) = 1.1\times 10^{-5}$ for the branching ratio. However, they also noted that final state interactions could lead to a larger value for the branching ratio $\mathrm{BR}(\rho^0 \to \pi^0\pi^0\gamma)$ through the mechanism $\rho^0 \to (\pi^+\pi^-)\gamma \to (\pi^0\pi^0)\gamma$. Bramon *et al.* [7] later considered the radiative vector meson decays within the framework of chiral effective Lagrangians enlarged to include on-shell vector mesons using chiral perturbation theory, and they calculated the branching ratios for various decays at the one-loop level, including both $\pi\pi$ and $K\bar{K}$ intermediate loops. In this approach, the decay $\rho^0 \to \pi^0\pi^0\gamma$ proceeds mainly through the charged pion $(\pi^+\pi^-)$ loops, contribution of kaon-loops being three orders of magnitude smaller, resulting in the decay rate $\Gamma(\rho^0 \to \pi^0\pi^0\gamma)_\chi = 1.42$ keV which is of the same order of magnitude as the VMD contribution. The interference between the pion-loop contribution and the VMD amplitude turns out to be constructive leading to $\mathrm{BR}(\rho^0 \to \pi^0\pi^0\gamma)_{VMD+\chi} = 2.6\times 10^{-5}$. However, this value is still substantially smaller than the latest experimental result quoted above.

Since the experimental result for the branching ratio $\mathrm{BR}(\rho^0 \to \pi^0\pi^0\gamma)$ is almost nearly twice the theoretical value calculated using VMD and chiral-loop amplitudes, the mechanism of the decay $\rho^0 \to \pi^0\pi^0\gamma$ should reexamined and additional contributions should be investigated. One additional contribution to the decay may be provided by the amplitude involving scalar-isoscalar σ meson as an intermediate state. Although the existence of the σ meson has long been controversial, an increasing number of theoretical and experimental analyses find a σ-pole position near $(500 - i250)$ MeV [8]. Furthermore, the $D^+ \to \sigma\pi^0 \to 3\pi$ decay channel observed by the Fermilab (E791) Collaboration is interpreted to provide direct experimental evidence for the σ meson where it is seen as a clear dominant peak with $M_\sigma = (478^{+24}_{-23} \pm 17)$ MeV, and $\Gamma_\sigma = (324^{+42}_{-40} \pm 21)$ MeV [9]. Since the σ meson is assumed to couple strongly to low mass pion pairs, the $\rho^0 \to \pi\pi\gamma$ decays thus provide an opportunity to investigate the theoretical ideas about the role the σ meson plays in the dynamics of low energy meson physics.

One way to include the effect of the σ meson in the decay mechanisms of $\rho^0 \to \pi^+\pi^-\gamma$ and $\rho^0 \to \pi^0\pi^0\gamma$ decays is to consider its contribution as resulting from a σ-pole intermediate state, that is to assume that the contributions of σ meson to the decay mechanisms of these decays result from the corresponding amplitudes of $\rho^0 \to (\sigma\gamma) \to (\pi^+\pi^-)\gamma$ and $\rho^0 \to (\sigma\gamma) \to (\pi^0\pi^0)\gamma$ reactions. In a previous work [10], two of the present authors calculated the branching ratio $\mathrm{BR}(\rho^0 \to \pi^+\pi^-\gamma)$ in a phenomenological framework using the pion bremsstrahlung amplitude and the σ-meson pole amplitude. The experimental value of this branching ratio was then used to calculate the coupling constant $g_{\rho\sigma\gamma}$ as a function of σ-meson parameters M_σ and Γ_σ. These authors in a following work [11] calculated the branching ratio $\mathrm{BR}(\rho^0 \to \pi^0\pi^0\gamma)$ using the values of the coupling constant $g_{\rho\sigma\gamma}$ thus obtained in a phenomenological approach where the contributions of σ-meson, *w*-meson intermediate states and of the pion loops are considered. The branching ratio $\mathrm{BR}(\rho^0 \to \pi^0\pi^0\gamma)$ obtained this way for $M_\sigma = 478$ MeV and

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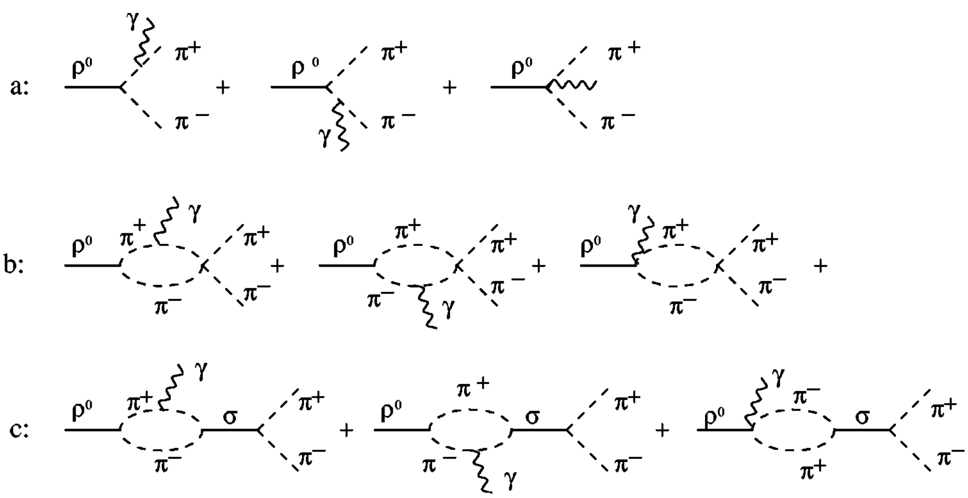

FIG. 1. Feynman diagrams for the decay $\rho^0 \rightarrow \pi^+ \pi^- \gamma$.

$\Gamma_\sigma = 324$ MeV was more than an order of magnitude larger than the experimental value. This unrealistic value was the result of the constant $\rho \rightarrow \sigma\gamma$ amplitude used and consequently the large coupling constant $g_{\rho\sigma\gamma}$ extracted from the experimental value of the branching ratio of the $\rho^0 \rightarrow \pi^+ \pi^- \gamma$ decay. Therefore, it may be concluded that it is not realistic to include the σ meson in the mechanisms of the radiative ρ^0-meson decays as an intermediate pole state.

On the other hand, $\rho^0 \rightarrow \pi^0 \pi^0 \gamma$ decay was also considered by Marco *et al.* [12] in the framework of unitarized chiral perturbation theory. They noted that the energies of two-pion system are quite large so that the decay cannot be treated with standard chiral perturbation theory. They used the techniques of chiral unitary theory developed earlier; a review is given by Oller *et al.* [13], to include the final state interactions of two pions by summing the pion loops through the Bethe-Salpeter equation. The branching ratio for $\rho^0 \rightarrow \pi^0 \pi^0 \gamma$ they obtained was $\mathrm{BR}(\rho^0 \rightarrow \pi^0 \pi^0 \gamma) = 1.4 \times 10^{-5}$ and they, furthermore, noted that this result could be interpreted as the result of the mechanism $\rho^0 \rightarrow (\sigma)\gamma \rightarrow (\pi^0 \pi^0)\gamma$ since $\pi^0 \pi^0$ interaction is dominated by the σ pole in the relevant energy regime of this decay.

Thus, it seems that a natural way to include the effects of the σ meson in the mechanisms of radiative ρ^0 meson decays is to assume that the σ meson couples to the ρ^0 meson through the pion loop. In this work, we reconsider the approach used in the Refs. [10] and [11], and we study the contribution of the σ-meson intermediate state amplitude to $\rho^0 \rightarrow \pi^+ \pi^- \gamma$ and $\rho^0 \rightarrow \pi^0 \pi^0 \gamma$ decays. We follow a phenomenological approach and assume that the σ meson couples to the ρ^0 meson through a pion loop; in other words, we assume that the amplitude $\rho^0 \rightarrow \sigma\gamma$ results from the sequential $\rho^0 \rightarrow (\pi^+ \pi^-)\gamma \rightarrow \sigma\gamma$ mechanism as suggested by the unitarized chiral perturbation theory in which the σ meson is generated dynamically by unitarizing the one-loop pion amplitudes. We use the coupling constants that are determined from the experimental values of the relevant quantities that we calculate employing the effective Lagrangians of our approach. Although the decay $\rho^0 \rightarrow \pi^+ \pi^- \gamma$ is dominated by the pion-bremsstrahlung amplitude, for the consistency of our formalism we study both of $\rho^0 \rightarrow \pi^+ \pi^- \gamma$ and $\rho^0 \rightarrow \pi^0 \pi^0 \gamma$ decays and calculate their branching ratios.

Our calculation is based on the Feynman diagrams shown in Fig. 1 for $\rho^0 \rightarrow \pi^+ \pi^- \gamma$ decay and in Fig. 2 for $\rho^0 \rightarrow \pi^0 \pi^0 \gamma$ decay. The last diagrams in Figs. 1(a),(b),(c) and in Figs. 2(b), (c) are the direct terms required to establish the gauge invariance. We describe the $\omega\rho\pi$ vertex by the effective Lagrangian

$$\mathcal{L}^{eff}_{\rho\omega\pi} = g_{\omega\rho\pi} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \omega_\nu \partial_\alpha \vec{\rho}_\beta \cdot \vec{\pi}, \qquad (1)$$

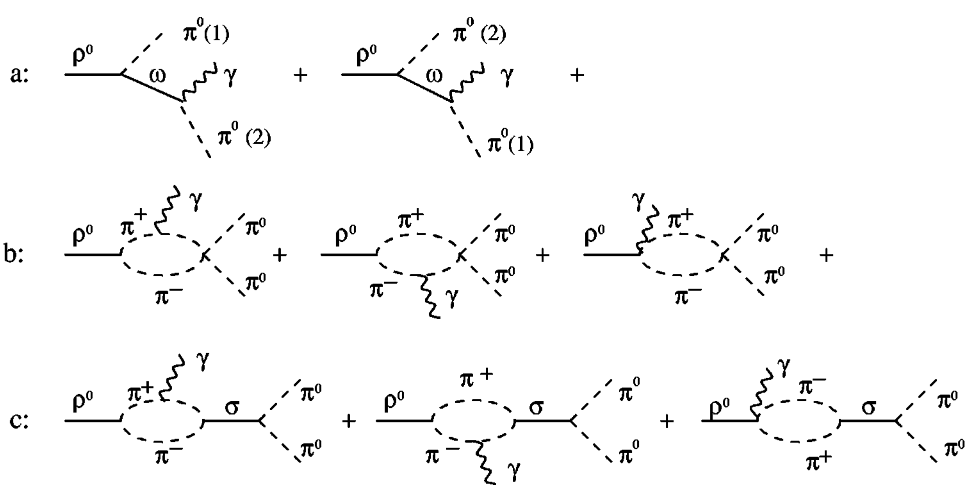

FIG. 2. Feynman diagrams for the decay $\rho^0 \rightarrow \pi^0 \pi^0 \gamma$.

which also conventionally defines the coupling constant $g_{\omega\rho\pi}$. Achasov *et al.* [14] assumed that $\omega\to 3\pi$ decay proceeds with the intermediate $\rho\pi$ state as $\omega\to(\rho)\pi\to\pi\pi\pi$ and using experimental value of the $\omega\to 3\pi$ width they determined this coupling constant as $g_{\rho\omega\pi}=(14.3\pm 0.2)$ GeV^{-1}. The $\rho\pi\pi$ vertex is described by the effective Lagrangian

$$\mathcal{L}^{eff}_{\rho\pi\pi}=g_{\rho\pi\pi}\vec{\rho}_\mu\cdot(\partial^\mu\vec{\pi}\times\vec{\pi}). \tag{2}$$

The experimental decay width of the decay $\rho\to\pi\pi$ [4] then yields the value $g_{\rho\pi\pi}=(6.03\pm 0.02)$ for the coupling constant $g_{\rho\pi\pi}$. For the $\sigma\pi\pi$ vertex we use the effective Lagrangian

$$\mathcal{L}^{eff}_{\sigma\pi\pi}=\frac{1}{2}g_{\sigma\pi\pi}M_\sigma\vec{\pi}\cdot\vec{\pi}\sigma. \tag{3}$$

Using the experimental values for M_σ and Γ_σ [9] as $M_\sigma=(483\pm 31)$ MeV and $\Gamma_\sigma=(338\pm 48)$ MeV, where statistical and systematic errors are added in quadrature [15], we obtain the strong coupling constant $g_{\sigma\pi\pi}$ as $g_{\sigma\pi\pi}=(5.34\pm 0.55)$. We note that our effective Lagrangians $\mathcal{L}^{eff}_{\sigma\pi\pi}$ and $\mathcal{L}^{eff}_{\rho\pi\pi}$ are the ones that result from an extension of the σ model to include the isovector ρ through a Yang-Mills local gauge theory based on isospin with the vector meson mass generated through the Higgs mechanism [16]. The $\omega\pi\gamma$ vertex is described by the effective Lagrangian

$$\mathcal{L}^{eff}_{\omega\pi\gamma}=g_{\omega\pi\gamma}\epsilon^{\mu\nu\alpha\beta}\partial_\mu\omega_\nu\partial_\alpha A_\beta\pi. \tag{4}$$

We then obtain the coupling constant $g_{\omega\pi\gamma}$ from the experimental partial width [4] of the radiative decay $\omega\to\pi^0\gamma$ as $g_{\omega\pi\gamma}=(0.706\pm 0.021)$ GeV^{-1}.

Meson-meson interactions were studied by Oller and Oset [17] using the standard chiral Lagrangian in the lowest order of chiral perturbation theory that contains the most general low energy interactions of the pseudoscalar meson octet in this order. We use their results for the four pseudoscalar amplitudes $\pi^+\pi^-\to\pi^0\pi^0$ and $\pi^+\pi^-\to\pi^+\pi^-$ that we need in the loop diagrams in Fig. 1(b) and in Fig. 2(b). We note that as shown by Oller [18] due to gauge invariance the off-shell parts of the amplitudes, which should be kept inside the loop integration, do not contribute, and as a result the amplitudes $\mathcal{M}_\chi(\pi^+\pi^-\to\pi^0\pi^0)$ and $\mathcal{M}_\chi(\pi^+\pi^-\to\pi^+\pi^-)$ factorize in the expressions for the loop diagrams.

In our calculation of the invariant amplitude, in the σ-meson propagator we make the replacement $q^2-M^2\to q^2-M^2+iM\Gamma$ and we use the energy-dependent width for the σ-meson which is given as

$$\Gamma_\sigma(q^2)=\Gamma_\sigma\frac{M_\sigma^2}{q^2}\sqrt{\frac{q^2-4M_\pi^2}{M_\sigma^2-4M_\pi^2}}\theta(q^2-4M_\pi^2). \tag{5}$$

For the loop integrals appearing in Figs. 1 and 2 we use the results of Lucio and Pestiau [19] who evaluated similar integrals using dimensional regularization. In our case the contribution of the pion-loop amplitude corresponding to the $\rho^0\to(\pi^+\pi^-)\gamma\to\pi^0\pi^0\gamma$ reaction can be written as

$$\mathcal{M}_\pi=-\frac{eg_{\rho\pi\pi}\mathcal{M}_\chi(\pi^+\pi^-\to\pi^0\pi^0)}{2\pi^2M_\pi^2}I(a,b)[(p\cdot k)(\epsilon\cdot u)-(p\cdot\epsilon)(k\cdot u)], \tag{6}$$

where $a=M_\rho^2/M_\pi^2$, $b=(p-k)^2/M_\pi^2$, $\mathcal{M}_\chi(\pi^+\pi^-\to\pi^0\pi^0)=-(s-M_\pi^2)/f_\pi^2$, $s=M^2_{\pi^0\pi^0}$, $f_\pi=92.4$ MeV, $p(u)$, and $k(\epsilon)$ being the momentum (polarization vector) of the ρ meson and photon, respectively. The amplitudes corresponding to $\rho^0\to(\pi^+\pi^-)\gamma\to\pi^+\pi^-\gamma$, $\rho^0\to(\pi^+\pi^-)\gamma\sigma\to\gamma\pi^+\pi^-$, and $\rho^0\to(\pi^+\pi^-)\gamma\sigma\to\gamma\pi^0\pi^0$ reactions can similarly be written. The function $I(a,b)$ is given as

$$I(a,b)=\frac{1}{2(a-b)}-\frac{2}{(a-b)^2}\left[f\left(\frac{1}{b}\right)-f\left(\frac{1}{a}\right)\right]+\frac{a}{(a-b)^2}\left[g\left(\frac{1}{b}\right)-g\left(\frac{1}{a}\right)\right] \tag{7}$$

where

$$f(x)=\begin{cases}-\left[\arcsin\left(\frac{1}{2\sqrt{x}}\right)\right]^2, & x>\frac{1}{4}\\ \frac{1}{4}\left[\ln\left(\frac{\eta_+}{\eta_-}\right)-i\pi\right]^2, & x<\frac{1}{4}\end{cases}$$

$$g(x)=\begin{cases}(4x-1)^{1/2}\arcsin\left(\frac{1}{2\sqrt{x}}\right), & x>\frac{1}{4},\\ \frac{1}{2}(1-4x)^{1/2}\left[\ln\left(\frac{\eta_+}{\eta_-}\right)-i\pi\right], & x<\frac{1}{4},\end{cases} \tag{8}$$

$$\eta_\pm=\frac{1}{2x}[1\pm(1-4x)^{1/2}].$$

We then calculate the invariant amplitude $\mathcal{M}$ (E_γ,E_1) from the corresponding Feynman diagrams shown in Figs. 1 and 2 for the decays $\rho^0\to\pi^+\pi^-\gamma$ and $\rho^0\to\pi^0\pi^0\gamma$, respectively. The differential decay probability for an unpolarized ρ^0 meson at rest is then given as

$$\frac{d\Gamma}{dE_\gamma dE_1}=\frac{1}{(2\pi)^3}\frac{1}{8M_\rho}|\mathcal{M}|^2, \tag{9}$$

where E_γ and E_1 are the photon and pion energies, respectively. We perform an average over the spin states of the ρ^0 meson and a sum over the polarization states of the photon. The decay width is then obtained by integration

$$\Gamma=\left(\frac{1}{2}\right)\int_{E_{\gamma,min}}^{E_{\gamma,max}}dE_\gamma\int_{E_{1,min}}^{E_{1,max}}dE_1\frac{d\Gamma}{dE_\gamma dE_1}, \tag{10}$$

where the factor $(\frac{1}{2})$ is included for the calculation of the decay rate for $\rho^0\to\pi^0\pi^0\gamma$ because of the $\pi^0\pi^0$ in the final state. The minimum photon energy is $E_{\gamma,min}=0$ and the maximum photon energy is given as $E_{\gamma,max}=(M_\rho^2$

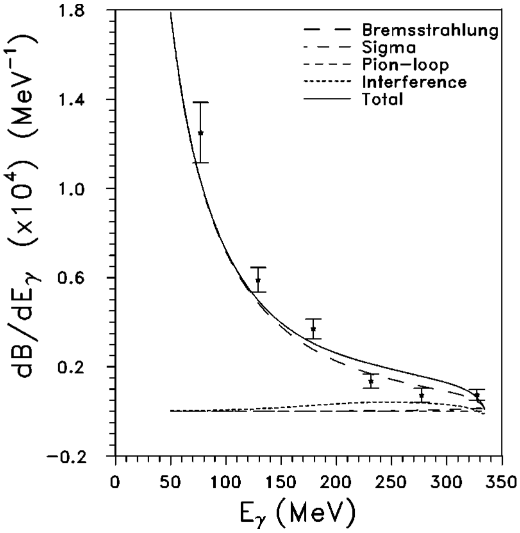

FIG. 3. The photon spectra for the branching ratio of $\rho^0 \to \pi^+\pi^-\gamma$ decay. The contributions of different terms are indicated. The experimental data taken from Ref. [3] are normalized to our results.

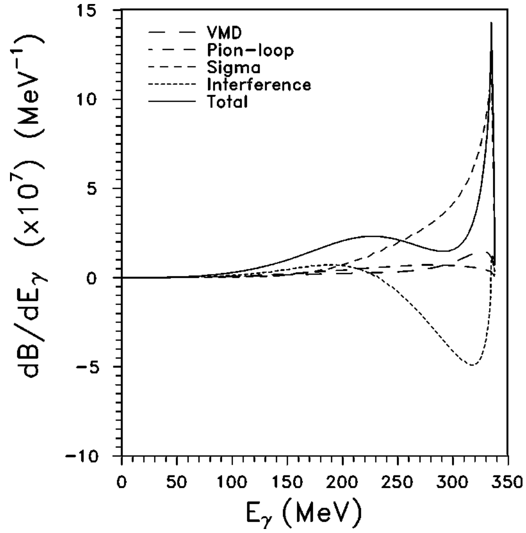

FIG. 4. The photon spectra for the branching ratio of $\rho^0 \to \pi^0\pi^0\gamma$ decay. The contributions of different terms are indicated.

$-4M_\pi^2)/2M_\rho = 338$ MeV. The maximum and minimum values for pion energy E_1 are given by

$$\frac{1}{2(2E_\gamma M_\rho - M_\rho^2)}[-2E_\gamma^2 M_\rho + 3E_\gamma M_\rho^2 - M_\rho^3 \pm E_\gamma \sqrt{(-2E_\gamma M_\rho + M_\rho^2)(-2E_\gamma M_\rho + M_\rho^2 - 4M_\pi^2)}].$$

The photon spectra for the branching ratio of the decay $\rho^0 \to \pi^+\pi^-\gamma$ are plotted in Fig. 3 as a function of photon energy E_γ. The contributions of bremsstrahlung and structural radiation amplitudes calculated with the pion loop and with the σ-meson intermediate state as well as the contribution of the interference term are shown as a function of the photon energy, where the minimum photon energy is taken as $E_{\gamma,min} = 50$ MeV since the experimental value of the branching ratio is determined for this range of photon energies [3]. In the same figure we also show the experimental data points taken from Ref. [3] which are normalized to our results. As it can be seen in Fig. 3, the shape of the photon energy distribution is reproduced well. As expected, the main contribution to the branching ratio comes from the pion-bremsstrahlung amplitude, the contributions of pion-loop and σ-meson intermediate states becoming noticeable only in the region of high photon energies. On the other hand, if a σ-meson pole model or equivalently a constant $\rho \to \sigma\gamma$ amplitude is used as in Ref. [10] the contribution of the σ term becomes increasingly important in the region of high photon energies dominating the contribution of the bremsstrahlung amplitude, and although its contribution is somewhat reduced by the interference term the σ-meson amplitude makes the main contribution to the branching ratio in the region of high photon energies conflicting with the experimental spectrum. For the contribution of different amplitudes to the branching ratio, we obtain $\mathrm{BR}(\rho^0 \to \pi^+\pi^-\gamma)_\gamma = (1.14 \pm 0.01) \times 10^{-2}$ from the bremsstrahlung amplitude, $\mathrm{BR}(\rho^0 \to \pi^+\pi^-\gamma)_\pi = (0.45 \pm 0.08) \times 10^{-5}$ from the pion-loop amplitude, and $\mathrm{BR}(\rho^0 \to \pi^+\pi^-\gamma)_\sigma = (0.83 \pm 0.16) \times 10^{-4}$ from the σ-meson intermediate state. If we consider also the interference between the π-loop and σ-meson amplitudes, we then obtain the contribution coming from the structural radiation as $\mathrm{BR}(\rho^0 \to \pi^+\pi^-\gamma) = (0.83 \pm 0.14) \times 10^{-4}$ which is in reasonable agreement with the experimental limit $\mathrm{BR}(\rho^0 \to \pi^+\pi^-\gamma) < 5 \times 10^{-3}$ deduced by Dolinsky *et al.* [3] for the structural radiation. Dolinsky *et al.* [3] also extracted the experimental limit $\mathrm{BR}[\rho^0 \to \epsilon(700)\gamma \to \pi^+\pi^-\gamma] < 4 \times 10^{-4}$ where the transition proceeds through the intermediate scalar resonance which is considerably lower. Our result for the contribution of the σ-meson intermediate state $\mathrm{BR}(\rho^0 \to \pi^+\pi^-\gamma)_\sigma = (0.83 \pm 0.16) \times 10^{-4}$ is also in good agreement with this experimental limit. For the total contribution, we obtain the branching ratio of the decay $\rho^0 \to \pi^+\pi^-\gamma$ as $\mathrm{BR}(\rho^0 \to \pi^+\pi^-\gamma) = (1.22 \pm 0.02) \times 10^{-2}$ for $E_\gamma > 50$ MeV. Our result is in reasonably good agreement with the experimental number $\mathrm{BR}(\rho^0 \to \pi^+\pi^-\gamma) = (0.99 \pm 0.16) \times 10^{-2}$ [3].

The photon spectra resulting from our calculation for the branching ratio of the $\rho^0 \to \pi^0\pi^0\gamma$ decay is shown in Fig. 4. The contributions of the VMD amplitude, the pion-loop amplitude, and the σ-meson intermediate state amplitude as well as the total interference term are indicated. The contributions of the VMD amplitude, pion-loop amplitude and σ-meson intermediate state amplitude to the branching ratio of the decay are $\mathrm{BR}(\rho^0 \to \pi^0\pi^0\gamma)_{VMD} = (1.03 \pm 0.02) \times 10^{-5}$, $\mathrm{BR}(\rho^0 \to \pi^0\pi^0\gamma)_\pi = (1.07 \pm 0.02) \times 10^{-5}$, and $\mathrm{BR}(\rho^0 \to \pi^0\pi^0\gamma)_\sigma = (4.96 \pm 0.18) \times 10^{-5}$, respectively. We see that the σ-meson intermediate state makes an important contribution to the branching ratio comparable to the contributions of VMD and pion-loop amplitudes. The values we obtain for $\mathrm{BR}(\rho^0 \to \pi^0\pi^0\gamma)_{VMD}$ and for $\mathrm{BR}(\rho^0 \to \pi^0\pi^0\gamma)_\pi$ are in agreement with previous calculations [6,7]. For the total branching ratio, including interference terms, we obtain the result $\mathrm{BR}(\rho^0\to\pi^0\pi^0\gamma)=(4.95\pm0.82)\times10^{-5}$ which is in a better agreement with the experimental result $\mathrm{BR}(\rho^0\to\pi^0\pi^0\gamma)=(4.1^{+1.0}_{-0.9}\pm0.3)\times10^{-5}$ [1] than the theoretical value obtained by using only VMD and chiral pion-loop amplitudes [7].

In our work, we follow a phenomenological approach and in a model for the decay mechanism of ρ^0-meson radiative decays, including the contribution coming from the σ-meson intermediate state as well as VMD and chiral pion-loop contributions, we calculate the branching ratios for the $\rho^0\to\pi^+\pi^-\gamma$ and $\rho^0\to\pi^0\pi^0\gamma$ decays. In our calculations of the branching ratios, the coupling constants that we use in our model are determined from the relevant experimental quantities. Our results for the branching ratios are in good agreement with the experimental values, and we believe that our study demonstrates that the contribution coming from the σ-meson intermediate state amplitude should be included in the analysis of radiative ρ^0-meson decays and, moreover, the σ meson should be considered to couple to the ρ^0 meson through a pion loop.

In a recent paper, Palomar *et al.* [20] also evaluated the branching ratios of the radiative ρ^0 and ω decays into $\pi^0\pi^0$ and $\pi^0\eta$. They used the sequential vector decay mechanisms in addition to chiral loops and ρ-ω mixing. For the $\rho^0\to\pi^0\pi^0\gamma$ decay ρ-ω mixing was negligible, but the branching ratio obtained with the sum of the sequential and loop mechanisms was about three times larger than with either mechanism alone, leading to a result $\mathrm{BR}(\rho^0\to\pi^0\pi^0\gamma)=4.2\times10^{-5}$ comparable with the present experimental value.

Therefore, in order to understand the mechanism of the $\rho^0\to\pi^0\pi^0\gamma$ decay and to obtain insight into the nature and the properties of the σ meson, and the role it plays in the dynamics of low energy meson physics, further experimental tests such as the measurements of invariant mass distributions will be very valuable.

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